Required Capital for Long-run Risks*

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Abstract

One of the objectives of the recent prudential regulation is to separate the computation of required capital for short- and long-run risks. This paper provides a coherent framework to define, compute, and update these components. We provide different examples, among which is the transition to low carbon economies.

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1 Introduction

An important objective of the recent prudential regulation is to separate the computation of required capital for short- and long-run risks (see, e.g. European Banking Authority, 2019). While the notion of long-run risks is not clearly defined, it encompasses several situations: the long-term holding of illiquid assets; the risk of a restructuration of a production process, for instance due to new rules on carbon issuing; the risk associated with depollution costs when dismantling a nuclear power plant in 2040; more generally the Environmental, Social and Governance (ESG) risks, or the pension savings with guaranted capital paid at the retirement age.

The standard approach to compute required capital in Basel 3 is based on a computation of a Value-at-Risk (VaR), or an Expected Shortfall (ES), at a short horizon—typically one year—and a critical level $\alpha = 5\%$, say (see Appendix 1 for a discussion of the VaR).[1] This approach might be directly extended to a long horizon, but this extension is not relevant for two reasons.[2] The first reason is that the risk to be covered by the required capital is usually large,[3] and the standard prudential approach—based for instance on short term VaR—assumes that it has to be covered at all dates before this risk is realized. This implies a large cost of required capital and a lack of investment against long term risks. Typically, this approach is not compatible with the transition to low carbon emissions and climate change adjustment. Thus, "imposing liquidity requirements would likely produce a reallocation of investments towards liquid shorter-term assets, while low carbon initiatives (for instance) require long term credit" (Campiglio, 2016, Section 5). The second reason is that long-run risks are usually not traded on financial markets, not well measured, and therefore largely unhedgeable. They have been widely neglected in the past and are not reflected in historical data. This implies that the conditional distributions of these risks are difficult to approximate as required in the standard VaR approach.

This paper introduces operational definitions of required capital which are more appropriate for a long term risk potentially realized at a given large maturity. For prudential supervision, this has to be done at the "individual" level (corporate, bank, or contract for pension saving), not at the macro level. The main idea is to define a progressive profile of required capital up to this maturity, and to avoid asking for a perfect hedge at all intermediate dates. This profile involves four ingredients: (i) the updating frequency, (ii) a regulatory discount rate, (iii) a design for the

[1] The horizon is of 3 to 5 years for the stress tests.

[2] See e.g. Dietz et al. (2016) for a climate Value-at-Risk based on an Integrated Assessment Model (IAM).

[3] For instance, the costs of demantling the French nuclear power plant park is assessed to be between €70bn and €100bn.

evolution of the reserves (which can be used to manage the interest of investors in long term risks), and (iv) a benchmark profile for the underlying risk, that is not market based.

The profile of the required capital is introduced in Section 2. We first consider the case of deterministic loss or profit at maturity and discuss the reserve evolution design as well as the discounting. Then, we extend the analysis to a stochastic asset value. In Section 3, we explain how to treat jointly short- and long-run risk factors. These long-run factors are based on a new class of processes called Ultra Long Run (ULR) processes. We examine in particular the long-run transition risks between two economies, the transition pertaining to the production process. Illustrations are provided in Section 4. Section 5 concludes. The discussion is completed by appendices: the first appendix reviews the standard Value-at-Risk approach (Appendix A); the second derives the long-run approximation of the distribution of cumulated profit (Appendix B).

2 The Design of Required Capital

Let us denote by t the first date at which the potential long-run risk is considered and T the maturity, i.e. the final date at which the risk is completely realized. The maturity T is fixed in the analysis. The intermediate dates are denoted $t+h, h=0,\dots,H$, where $H=T-t$ is the initial time-to-maturity. This initial time-to-maturity is large, typically between 10 and 50 years. This horizon is often beyond the traditional horizons of most actors, imposing a cost on future generations that the current generation may not endure. That includes the horizon of the central banks and supervisors, who are bound by their mandates (Carney, 2015).[4] In this section, we consider that there is a potential loss at maturity, denoted $X_{t+H}=X_T$, and that this loss has to be hedged by a sequence of Required Capital Calls RCC_{t+h}, $h=1,\dots,H$. In Section 3, we will also consider the case where X_{t+H} is a gain, more precisely a cumulated profit. This case could be covered by keeping the definition of X_{t+H} as a loss, the gain being $-X_{t+H}$, and the $-RCC_{t+h}$, $h=1,\dots,H$, being possibilities of investment. Alternatively, we can also define X_{t+H} as a gain, and in this case the RCC_{t+h} would directly be the possibilities of investment. The latter sign convention will be used in Sections 3.2, 3.3, and 4.

[4] See also Goodhart (2010), Batten et al. (2016), Campiglio et al. (2018), Dikau and Volz (2021), Bolton et al. (2020) for discussions of the proactivity and mandates of such institutions.

2.1 Deterministic Loss at Maturity

Let us consider the case of a deterministic amount X_{t+H}. Depending on the case under consideration, this amount has different interpretations: it is a reimbursement *in fine* in the case of a pension saving scheme with guaranteed capital; it is a cumulated sum of losses in the context of portfolios of contracts with different maturity dates; it reflects cumulated flows of expenses for the dismantling of nuclear power plants; it may also correspond to cumulated flows of negative profits, as will be the case in our carbon transition example (Subsection 3.3).

This final amount X_{t+H} is constituted progressively, by cumulating a sequence of "regular payments", or Required Capital Call: $RCC_{t+h}, h = 1,\ldots,H$.

Without discounting, several types of profile of RCC_{t+h} can be considered. Let us focus on exponential profiles:

$$RCC_{t+h} = \mu(t,H)\delta^{h-1},$$

where δ is positive. The monthly payments are constant if $\delta = 1$, increasing (respectively, decreasing), if $\delta > 1$ (resp. $\delta < 1$). The limiting cases $\delta = 0$ (resp. $\delta = \infty$) correspond to a required capital fixed constant equal to X_{t+H}, therefore to a total protection demanded since date $t+1$ (resp. to zero protection up to $t+H-1$, and a complete protection at $t+H$). Increasing profile implies less effort of reimbursment (or reserve constitution) in the short run. The importance of such profile is easily understood for the application to pension saving with guaranteed capital paid at retirement age: the fund receives regular payments by the individual and can invest them in more or less risky assets. By fixing $\delta > 1$, the supervision is imposing a minimal proportion invested in a (non remunerated) riskfree asset, and this proportion increases when getting closer to the maturity date to protect the future pensioner. This prudential instrument is in fact, if not in name, a form of credit guidance to monitor the credit allocation between short and long-run risks.[5]

In order to get the amount X_{t+H} at maturity, we need $RC_{t+H} = X_{t+H}$. Therefore, since

$$RC_{t+H} = \sum_{h=1}^{H} RCC_{t+h} = \mu(t,H)\frac{1-\delta^H}{1-\delta},$$

[5]See Bezemer et al. (2018) for a discussion of credit guidance policies. Largely abandoned in the 1980's with the argument that they can distort the efficient allocation of resources, they are put in place to develop priority sectors and help innovation. Relatedly, a growing literature investigates how monetary policy, through the choice of eligible collateral and/or asset purchase programs (Quantitative Easing) can affect the relative costs of green/brown investments [see, e.g., Matikainen et al. (2017), de Grauwe (2019), and Papoutsi et al. (2020)].

we need

$$\mu(t,H) = X_{t+H}\frac{1-\delta}{1-\delta^H} \quad \text{for} \quad \delta \neq 1, \quad \text{and} \quad \mu(t,H) = X_{t+H}/H, \quad \text{if} \quad \delta = 1. \tag{2.1}$$

In the standard supervision, there is no remuneration of the reserves of corporates or banks under supervision. Indeed reserves are deposits that private banks and insurance companies hold at the central bank. Therefore the total required capital at $t+h$ is:

$$RC_{t+h} = \sum_{k=1}^{h} RCC_{t+k} = X_{t+H}\frac{1-\delta^h}{1-\delta^H}, \tag{2.2}$$

and the remaining balance is:

$$B_{t+h} = X_{t+H} - RC_{t+h}. \tag{2.3}$$

These formulas can be rewritten as updating formulas.

Lemma 1. *Under deterministic loss, no discounting, the exponential profile is such that:*

$$\begin{aligned} RCC_{t+h} = RC_{t+h} - RC_{t+h-1} &= \frac{1-\delta}{1-\delta^{H-h+1}}(X_{t+H} - RC_{t+h-1}), \quad h = 1,\dots,H, \\ &= \frac{1-\delta}{1-\delta^{H-h+1}} B_{t+h-1}, \end{aligned}$$

with $RC_t = 0$ and B_{t+h-1} is given by (2.3).

Proof. We have, by (2.2):

$$\begin{aligned} \frac{1-\delta}{1-\delta^{H-h+1}}(X_{t+H} - RC_{t+h-1}) &= \frac{1-\delta}{1-\delta^{H-h+1}}\left(X_{t+H} - X_{t+H}\frac{1-\delta^{h-1}}{1-\delta^H}\right) \\ &= X_{t+H}\frac{1-\delta}{1-\delta^{H-h+1}}\frac{\delta^{h-1}-\delta^H}{1-\delta^H} = X_{t+H}\frac{1-\delta}{1-\delta^H}\delta^{h-1} \\ &= RCC_{t+h} = RC_{t+h} - RC_{t+h-1}, \end{aligned}$$

which gives the result. □

Lemma 1 shows that the required capital call at date $t+h$ is proportional to the remaining balance with a proportionality coefficient depending on the residual maturity $H-h$ and on the rate δ.

Possible evolutions of RC_{t+h} and RCC_{t+h}, for different values of δ, are illustrated in Figure 1. Panel b shows that the RCC_{t+h} are monotonously decreasing (resp. increasing) when $\delta < 1$ (resp. $\delta > 1$).

Other recursive formulas can also be derived such as:

$$RC_{t+h} - \delta RC_{t+h-1} = X_{t+H} \frac{1-\delta}{1-\delta^H}, \quad h = 1, \dots, H. \tag{2.4}$$

The adaptive formula in Lemma 1 is valid when there is no discounting. Let us now explain how to include a regulatory discounting. For expository purpose, let us consider an initial credit amount $\overline{X}_t$ and a regulatory rate, such that $X_{t+H} = \overline{X}_t(1+r)^H$. This regulatory rate is a supervisory instrument. It is not equal in general to the (long-run) market rate, to some model based rate adjusted for climate change or low carbon transition (Pindyck, 2013; Stern, 2016), or to a ceiling rate. The formula in Lemma 1 is easily adjusted by considering the "discounted" loss:

Definition 1. *Under deterministic loss and regulatory discounting, the exponentially weighted profile is such that:*

$$\begin{aligned} RCC_{t+h} = RC_{t+h} - RC_{t+h-1} &= \frac{1-\delta}{1-\delta^{H-h+1}} \left[\frac{X_{t+H}}{(1+r)^{H-h}} - RC_{t+h-1} \right], \quad \text{for } \delta \neq 1, \\ &= \frac{1}{H-h+1} \left[\frac{X_{t+H}}{(1+r)^{H-h}} - RC_{t+h-1} \right], \quad \text{if } \delta = 1, \end{aligned} \tag{2.5}$$

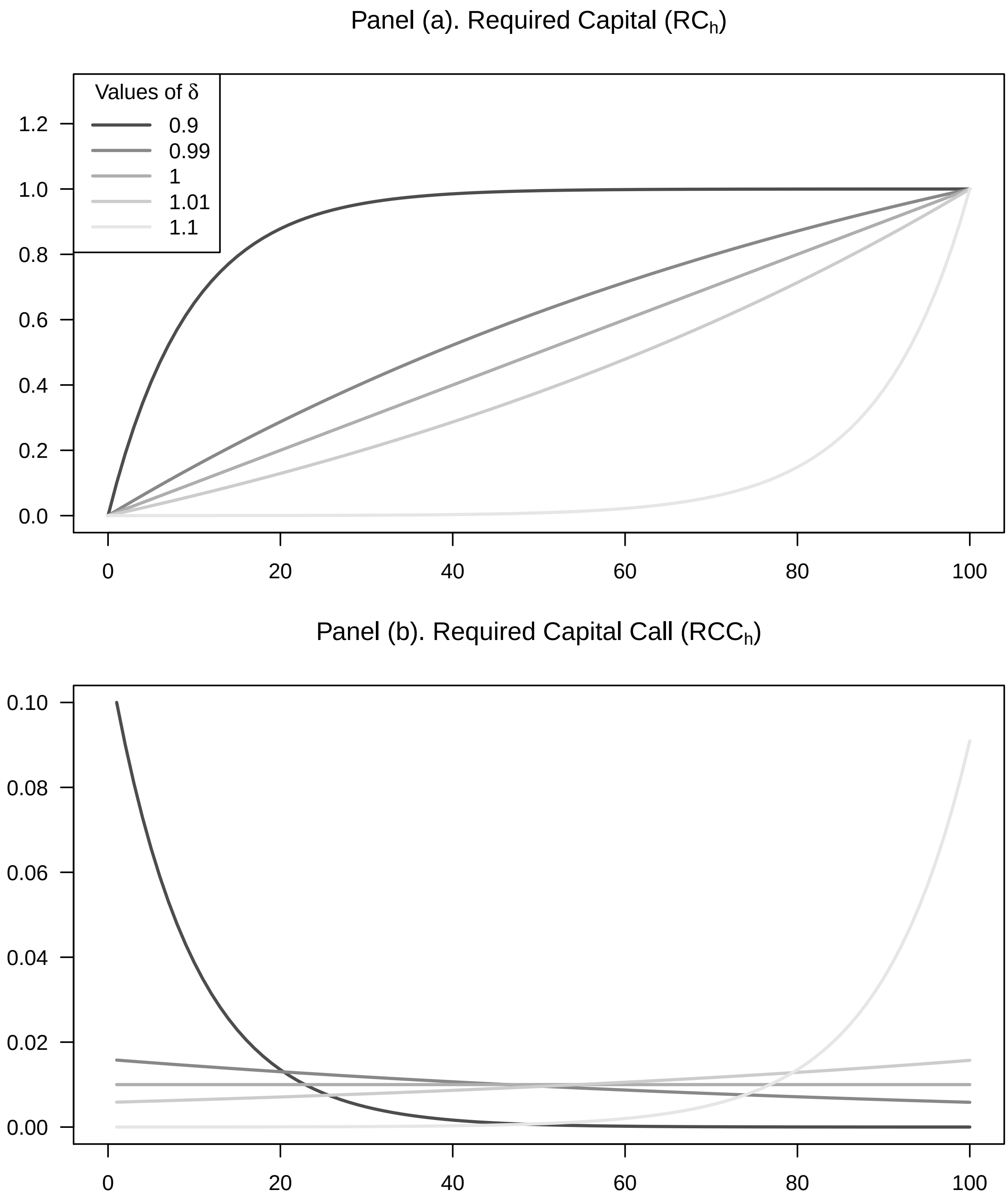
with $RC_t = 0$.

Formula (2.5) considers an objective $X_{t+H}/(1+r)^{H-h}$ at date $t+h$ and the residual exponential profiles associated with this objective. At date $t+h+1$, the objective is updated. The recursive equation (2.5) is easily be solved recursively. Figure 2 shows how the objective of the profile is updated with h.

Different discounting and remuneration schemes could be adopted.

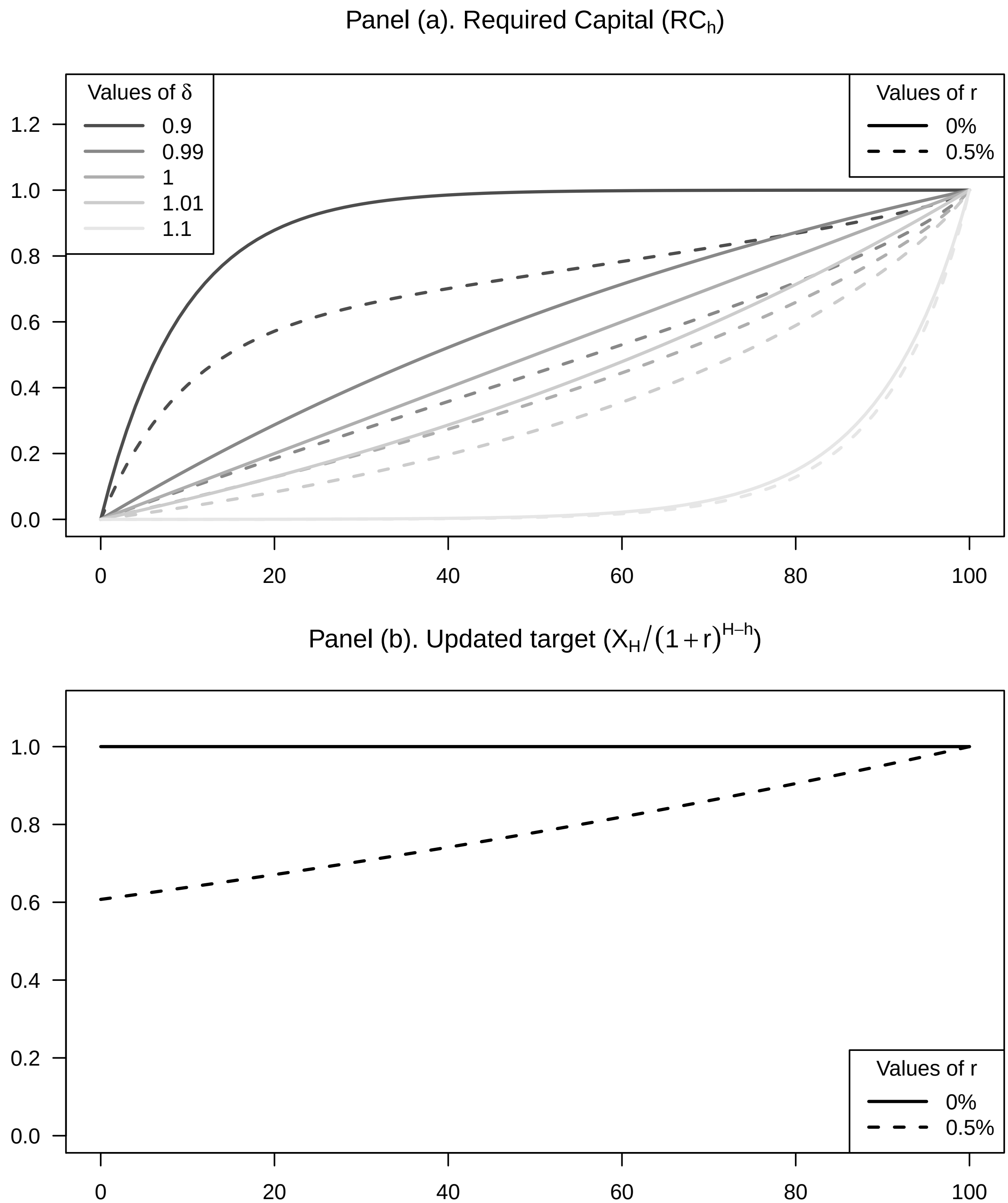
2.2 Stochastic Loss at Maturity

The updating formula (2.5) is the basis of an extension when the loss at maturity (X_{t+H}) is stochastic. This is for instance the case when X_{t+H} corresponds to the cost of restructuration, of depollution, or to a short-sell investment in an illiquid stock (without dividends).



Notes: This figure illustrates the updating formula given by Lemma 1. The initial date t is set to 0, and H is set to 100. The loss at maturity (X_H) is taken equal to 1. Panels (a) and (b) respectively display RC_h and RCC_h.

Figure 1: Influence of δ on the evolutions of RC_h and RCC_h



Notes: Panel (a) illustrates the influence of discounting (see Definition 1) on RC_h. The initial date t is set to 0, and H is set to 100. The loss at maturity (X_H) is taken equal to 1. Panel (b) displays the updated target, that is $X_H/(1+r)^{H-h}$.

Figure 2: Discounting of the target and required capital profiles

At each date $t+h$, we need a "valuation" $X^*_{t+h,t+H}$ of X_{t+H}.

Definition 2. *Under stochastic loss, regulatory discounting and sequence of valuations $X^*_{t+h,t+H}$, $h = 1,\dots,H$, with $X^*_{t+H,t+H} = X_{t+H}$, the profile of required capital is such that:*

$$RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}}\left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right], \tag{2.6}$$

with $RC_t = 0$.

This profile is perfectly defined once the supervisor has selected: (i) the regulatory (long-run) discount rate r; (ii) the rate δ of the exponential design of RCC; (iii) the sequence of benchmark "valuations" $X^*_{t+h,t+H}$, $h = 1,\dots,H$, of the risk at maturity.

Two remarks are in order. The first one is that the updating formula (2.6) does not ensure an increasing required capital. If necessary, we obtain an increasing pattern of RC_{t+h} by transforming it into:

$$RC_{t+h} - RC_{t+h-1} = \frac{1-\delta}{1-\delta^{H-h+1}}\left[\frac{X^*_{t+h,t+H}}{(1+r)^{H-h}} - RC_{t+h-1}\right]^+, \quad h = 1,\dots,H-1. \tag{2.7}$$

with $RC_t = 0$, and $X^+ = \max(X,0)$. This modification is in line with the standard formula for margin call in the definition of futures.[6]

A second remark pertains to the remuneration of the required capital. Usually, the amount of required capital is put into a special account, managed by an independent authority. In general, it is not remunerated, or is weakly remunerated for the institution under supervision. The authority—central bank or agency—may invest this amount in the riskfree assets proposed on financial markets, and use the interest to contribute to the financing of supervision. Again, the general approach presented here could be adjusted in order to reflect alternative remuneration strategies. In particular, it may depend on the sign of RC—a negative RC corresponding to a profit.

To make the RC profiles (2.6 or 2.7) operational, the sequence of benchmark valuations has to be specified. Different valuation schemes can be selected and chosen depending on the type of long-run risk.

[6] Note that the RCC's are not margin calls, since, the required capital RC_{t+h} is not sufficient to be totally protected against a complete default on X_{t+H} at an intermediate date $t+h$.

(i) **Mean-variance scheme**. In this context, we set:

$$X^*_{t+h,t+H} = \mathbb{E}_{t+h} X_{t+H} + \frac{A}{2} \mathbb{V}ar_{t+h} X_{t+H}, \tag{2.8}$$

where $\mathbb{E}_t(X_{t+H})$ (resp. $\mathbb{V}ar_t(X_{t+H})$) is an expected loss (resp. a loss variance), and A is the (absolute) risk aversion of the supervisor. This scheme is appropriate for the example of depollution cost, for which the costs and their uncertainties have to be regularly updated. They can decrease if new depollution techniques are introduced, increase if there are more severe constraints on the depollution level to be reached.

(ii) **Certainty equivalent scheme**. This approach demands a predictive distribution of the loss and a utility function $\mathscr{U}$. We then have:

$$X^*_{t+h,t+H} = \mathscr{U}^{-1}[\mathbb{E}_{t+h} \mathscr{U}(X_{t+H})]. \tag{2.9}$$

In the context of a CARA utility function, that is $-\exp(AX)$ when X is a loss, we get:

$$X^*_{t+h,t+H} = \frac{1}{A} \log \mathbb{E}_{t+h} \exp(AX_{t+H}), \tag{2.10}$$

where $A > 0$, is the absolute risk aversion of the supervisor.

If the conditional distribution of X_{t+H} is Gaussian, formula (2.10) reduces to the mean-variance scheme (2.8).

If X is a gain, the CARA function is $-\exp(-AX)$, and the quantity $X^*_{t+h,t+H}$ becomes $-\frac{1}{A} \log \mathbb{E}_{t+h} \exp(-AX_{t+H})$, that is $\mathbb{E}_{t+h}(X_{t+H}) - \frac{A}{2} \mathbb{V}ar_{t+h}(X_{t+H})$ in the Gaussian case.

(iii) **Risk-Neutral scheme**. The valuation is defined as:

$$X^*_{t+h,t+H} = \mathbb{E}^Q_{t+h}(X_{t+H}), \tag{2.11}$$

where Q is a distribution adjusted for risk, or risk-neutral distribution (under a zero riskfree rate). This valuation is the standard pricing formula when the "asset" corresponding to the loss (equal to X_{t+H} on date $t+H$) can be regularly traded between t and $t+H$ on competitive and highly-liquid markets. This is not the case for the long-run risks we are interested in.

Indeed, this approach implies the linearity of the valuation formula, that is:

$$\mathbb{E}^{Q}_{t+h}(\lambda X_{t+H}) = \lambda \mathbb{E}^{Q}_{t+h}(X_{t+H}), \quad \text{for any } \lambda > 0.$$

This major assumption also underlies the definition of coherent risk measures (see Artzner et al., 1999).

Consider the depollution cost. Is the cost of depollution for ten nuclear power plants equal to ten times the cost of depollution of a single plant? Likely not, since marginal costs may be decreasing (because of the economy of scale), or symmetrically increasing if there is some rationing in the number of specialized workers available at the same time. The same remark applies, if X_{t+H} represents the payment of a large short sell in an illiquid financial asset. The liquidation of this asset will likely be accompanied by penalties.

3 Long-run Transition Risks

Let us now discuss how to treat jointly the short- and long-run risks included in a portfolio and/or a balance sheet. We first propose an additive treatment of these risks. Then we consider the case where the portfolio allocation and/or the lines of the balance sheet depend jointly of short- and long-run risk factors in a nonlinear way [see the remark on nonlinearity in European Banking Authority (2020), point 93, and the literature on evolutionary models and complex dynamic systems, in particular Monasterolo et al. (2019), Cahen-Fourot et al. (2020)]. Finally, we consider the case where there is a long-run transition between two situations. A typical example is the transition between the current economy and a low-carbon one, this transition entailing a modification of the production process of the firms. Whereas the first two cases are based on pure reduced-form predictive models, the last case is more structural.

3.1 Additive Short and Long-run Risks

Let us consider the example of a pension scheme managed by a generation of contracts with the same maturity T. Between t and $t+H=T$, a pension fund regularly receives premiums from the individuals and can use them to invest in two types of assets: illiquid and more liquid ones. The latter ones can be used to feed the RC account.

At $t+h$ before the call and the premium payment

Asset	Liability
$\bar{A}_{t+h-1}$	X_{t+H}
RC_{t+h-1}	
	l_{t+h}
a_{t+h}	
A_{t+h}	L_{t+h}

At $t+h$ after the call and the premium payment

Asset	Liability
$\bar{A}_{t+h}$	X_{t+H}
RC_{t+h}	
rc_{t+h}	l_{t+h}
a^*_{t+h}	
A_{t+h}	L_{t+h}

Figure 3: Balance Sheets at $t+h$

Figure 3 represents the balance sheets of this closed fund at date $t+h$, before and after the capital calls. As usual we distinguish the frozen, i.e. totally illiquid, components of the balance sheets. These are the amount of guarantees X_{t+H} on the liability side, as well as the amount of future premiums $\bar{A}_{t+h}$ and the RC at the beginning of period $t+h$ on the asset side. The unfrozen components are denoted a_{t+h} and l_{t+h} on the asset and liability sides, respectively. For instance, l_{t+h} includes the management costs, such as salaries, whereas a_{t+h} includes financial asset holdings in more or less liquid assets. These are simplified balance sheets, assuming that there is no death of contractors before $t+H$, and that the individual contracts are not partly sold on a secundary market as insurance linked security (ILS).[7]

In this special application the amount of premiums payed in cash at $t+h$ is $\bar{A}_{t+h-1} - \bar{A}_{t+h}$. At the beginning of the period and if $\delta > 1$, these premiums can likely be sufficient to cover the capital calls. But this is no longer the case close to maturity, when some financial assets in a_{t+h} have to be sold to satisfy the capital requirement.

At date $t+h$, the unfrozen value $W_{t+h} = a_{t+h} - l_{t+h}$ has to be sufficiently large to satisfy the next call for long-run risk. Therefore a short-run required capital at $t+h-1$, denoted by $rc_{t+h-1}(\alpha)$, can be based on a level-α conditional VaR at horizon 1, such that:

$$\mathbb{P}_{t+h-1}[W_{t+h} + rc_{t+h-1}(\alpha) > RCC_{t+h}] = 1-\alpha, \tag{3.1}$$

where RCC_{t+h} is the long-run call defined in Section 2 and $\mathbb{P}_{t+h-1}$ denotes the probability conditional on the information available at date $t+h-1$. Then, the total RC at date $t+h$ is the sum of the short-run RC (i.e. rc_{t+h}), and the long-run RC (i.e. RC_{t+h}), whereas the total call is:

[7] For expository purpose, we do not discount X_{t+H} and the future sequence of premiums. This is usually done by actuarial techniques, using a contractual interest rate.

$RC_{t+h} - RC_{t+h-1} + rc_{t+h} = RCC_{t+h} + rc_{t+h}.$

3.2 Mixing Short and Long-run Factors

The portfolios of banks and insurance companies include assets whose values are driven by both short and long-run factors. To distinguish them, we can consider that short-run factors are highly volatile and mean reverting, i.e. without effects in the long run, whereas the long-run factors have highly persistent effects, but their changes are almost invisible in the short run. Climate change stands as a key example (see, e.g., IPCC, 2014; Carney, 2015; Campiglio et al., 2018; Nordhaus, 2019, for the challenges associated with climate change). Consequences of a low carbon policy are not visible at the daily frequency, but can become visible at a year frequency.

Climate-related factors have an effect on corporate results. They can increase the production costs and/or diminish the demand of the products. As a consequence, this may increase the probability of default and then diminish the value of the loans and of the stocks (see Industrial and Commercial Bank of China, 2016; Boermans and Galema, 2017; Thomä et al., 2017; Devulder and Lisack, 2020). At the limit a technology with a less damaging impact on the environment can replace a technology that is more damaging, hence making it obsolete. Climate change has also different geographical impacts (reflecting different beta's of climate change, see Kahn et al., 2019), and this heterogeneity has also to be taken into account, for instance for sovereign's bond portfolios (see Battiston et al., 2019).

Let us now consider an entity (corporate, bank, or portfolio) with a result, or profit, P_{t+h} at date $t+h$. Let us assume that this result depends on a short-run factor and a long-run one, these factors being respectively denoted by $y_{s,t+h}$ and $y_{l,t+h}$:

$$P_{t+h} = g(y_{s,t+h}, y_{l,t+h}), \tag{3.2}$$

where g may be nonlinear.

The cumulated result on the period is:

$$CP_{t+H} = \sum_{h=1}^{H} P_{t+h} = \sum_{h=1}^{H} g(y_{s,t+h}, y_{l,t+h}). \tag{3.3}$$

We expect the short-run factor to be highly volatile, but with a weak serial correlation, and the long-run factor to vary gradually, without significant visible impact in the short run.

Let us first consider the situation where the gradual evolution is around an "equilibrium" value y_l^*. The expression (3.3) can then be replaced with its first-order expansion with respect to y_l:

$$CP_{t+H} \simeq \sum_{h=1}^{H} g(y_{s,t+h}; y_l^*) + \sum_{h=1}^{H} (y_{l,t+h} - y_l^*) \frac{\partial g}{\partial y_l}(y_{s,t+h}; y_l^*). \tag{3.4}$$

Let us now discuss the stochastic behaviour of CP_{t+H} for large H, when the short- and long-run factors are independent, process $(y_{s,t})$ being a strictly stationary process and (y_{lt}) an Ultra Long-Run (ULR) component deduced from an Ornstein-Uhlenbeck process:

$$d\tilde{y}(\tau) = -k\tilde{y}(\tau)d\tau + \sqrt{2k}\, dW(\tau), \quad k > 0, \tag{3.5}$$

by a time deformation (see Gouriéroux and Jasiak, 2020):

$$y_{l,t} = \tilde{y}(t/H). \tag{3.6}$$

The ULR Component satisfies a discretized stationary Gaussian autoregressive process of order 1:

$$y_{l,t} = \rho_H y_{l,t-1} + \sqrt{1 - \rho_H^2}\, \varepsilon_{l,t}, \tag{3.7}$$

where $(\varepsilon_{l,t})$ is a Gaussian standard noise and $\rho_H = \exp(-k/H) < 1$. When H is large (tends to infinity), ρ_H tends to 1, and the trajectory of the ULR component is such that $y_{l,t} = y_{l,t-1}, \forall t$. Thus, (3.7) provides a stationary dynamics, close to a constant trajectory whose level is stochastic. Indeed the marginal distribution of $y_{l,t}$ is standard Gaussian and then independent of H.

We derive, in Appendix B.1, the limiting behaviour of CP_{t+H}:

$$\frac{1}{H} CP_{t+H} \simeq \mathbb{E}[g(y_{s,t}; y_l^*)] + \mathbb{E}\left[\frac{\partial g}{\partial y_l}(y_{s,t}; y_l^*)\right] \left(\int_0^1 \tilde{y}(u)du - y_l^*\right). \tag{3.8}$$

A similar analysis can be performed without applying a first-order expansion around an equilibrium value. Let us denote by f the stationary density of the noise $y_{s,t}$, and by G the function defined by:

$$G(y_l) = \int g(y_s, y_l) f(y_s) dy_s. \tag{3.9}$$

Then we get (see Appendix B):

$$\frac{1}{H} CP_{t+H} \simeq \int_0^1 G(\tilde{y}_u) du. \tag{3.10}$$

We see, from formulas (3.8) or (3.10), that, for large H, CP_{t+H} is stochastic through the long-run component $\tilde{y}$ only. It depends on the short-run component through its stationary, i.e., long-run, distribution. Therefore, we can apply the approach of Section 2 with $X_{t+H} = -CP_{t+H}$, or $X_{t+H} = CP_{t+H}$ by changing the sign convention, to derive the sequence of RCC_{t+h}, $h = 1, \ldots, H$. Then, at date $t+h-1$, the short-run required capital rc can be fixed as in (3.1) by:

$$\mathbb{P}_{t+h-1}[g(y_{s,t+h}, y^*_{l,t+h-1}) + rc_{t+h-1}(\alpha) > RCC_{t+h}] = 1 - \alpha.$$

The approach above is usually applied to different corporates $i = 1, \ldots, n$, say. Equation (3.2) then becomes:

$$P_{i,t+h} = g_i(y_{i,s,t+h}, y_{l,t+h}), \quad i = 1, \ldots, n,$$

where $y_{i,s}$ is an idiosyncratic factor, y_l a common long-run factor, and g_i depends on the corporate. This common (or systematic) factor creates the dependence between individual corporate risks.

3.3 Transition Risk

This section introduces a structural dynamic model describing the transition to a low carbon economy. In the model, firms are led to adjust their production process, and we examine the impact of this adjustment on their probability of default.[8] This structural analysis suggests that a prudential supervision does not only require the financial balance sheets, but also technical reports for the production function, including the carbon features. This is consistent with the Greenhouse Gas Emission Reports (GHGRP),[9] which include a carbon balance and are expected to be produced every year.

[8]This section concerns productive corporates and does not apply to financial institutions. The main reason is the assumption below of a fixed number of inputs, whereas a bank can increase the numbers and types of inputs by increasing the number and type of granted loans, say. That is, the modelling of this section is not appropriate to account for the creation of money by private banks.

[9]Such GHGRP include all Greenhouse Gas, as methan (H_4), nitrous oxyde ($N_2 0$), hydrofluorocarbon (HCF), perfluorinated hydrocarbons (PFC), sulfur hexafluoride (SF_6), not only CO_2. It can be extended to other environmental aspects as air pollutants, water stress, and various wastes (see, e.g., British Columbia (BC), 2015; European Banking Authority, 2020, Annex 1). For simplicity, we consider only carbon in the present model.

3.3.1 The standard production function

For expository purpose, we consider a firm with production function g. There are two inputs and one output:[10]

$$y = g(x_1, x_2), \tag{3.11}$$

where x_1 and x_2 are input quantities and y is the output quantity. Input prices are denoted by $p = (p_1, p_2)$, and π denotes the price of the output. Under standard assumptions, the producer is assumed to maximize her profit:

$$\max_{x_1,x_2} \pi g(x_1,x_2) - p_1 x_1 - p_2 x_2, \text{ s.t. } g(x_1,x_2) = y, \tag{3.12}$$

or, equivalently, to minimize her cost:

$$\min_{x_1,x_2} p_1 x_1 + p_2 x_2, \text{ s.t. } g(x_1,x_2) = y.$$

If the production function is differentiable, this leads to the first-order conditions:

$$\frac{\partial g(x_1,x_2)}{\partial x_j} = p_j/\lambda, j = 1,2, \tag{3.13}$$

where λ is a Lagrange multiplier in the cost minimization problem. Let us denote $\hat{x}_1, \hat{x}_2$ the solutions of (3.13). The solutions of (3.12) are:

$$\begin{cases} \hat{x}_1, \hat{x}_2 & \text{if } \pi g(\hat{x}_1,\hat{x}_2) - p_1\hat{x}_1 - p_2\hat{x}_2 > 0, \\ \text{no production,} & \text{otherwise.} \end{cases} \tag{3.14}$$

Example 1. *Strict complementary*

If $g(x_1,x_2) = \min(a_1x_1, a_2x_2)$*, with* $a_1 > 0$ *and* $a_2 > 0$*, then the optimum is:* $\hat{x}_1 = y/a_1$ *and* $\hat{x}_2 = y/a_2$*, if* $\pi - p_1/a_1 - p_2/a_2 > 0$*, and no production, otherwise.*

Note that $c_j = 1/a_j, j = 1,2$*, are technical coefficients usually given in input-output tables: to produce one unit of output, we need* c_1 *units of input 1 and* c_2 *units of input 2.*

[10] In practice the production process of the firm involves a large numbers of both inputs and outputs, leading to a large input-output table [see e.g. Timmer (2012) at the world level, Wilting and van Oorschot (2017) at a country level]. Defining precisely a standardized list of inputs/outputs is currently one main goal of the prudential supervision.

Example 2. *Substituability*

Substituable inputs can be represented by a Cobb-Douglas production function:

$$g(x_1,x_2) = Ax_1^{\alpha_1}x_2^{\alpha_2}, \quad \textit{with } A > 0,\ \alpha_1 > 0,\ \alpha_2 > 0.$$

The optimum is:

$$\hat{x}_1 = \left(\frac{y}{A}\right)^{\dfrac{1}{\alpha_1+\alpha_2}} \left(\frac{p_2}{p_1}\frac{\alpha_1}{\alpha_2}\right)^{\dfrac{\alpha_2}{\alpha_1+\alpha_2}},$$

$$\hat{x}_2 = \left(\frac{y}{A}\right)^{\dfrac{1}{\alpha_1+\alpha_2}} \left(\frac{p_1}{p_2}\frac{\alpha_2}{\alpha_1}\right)^{\dfrac{\alpha_1}{\alpha_1+\alpha_2}}.$$

3.3.2 Production function that includes "carbon"

Let us now explain how carbon can be included in the production function.[11] Carbon can be taken into account as an input as well as an output—leading possibly to a "circular economy". We obtain a multiple input-output production function:

$$\begin{cases} y = f_1(x_1,x_2;z_1), \\ z = f_2(x_1,x_2;z_1), \end{cases} \tag{3.15}$$

where z_1 (resp. z) is the quantity of carbon input (resp. carbon output). Let us denote by $-\pi_c$ the price of carbon emissions (the negative sign accounts for negative externalities), and by q the price of carbon inputs. For instance, in a macroanalysis, f_1 could be a three-factor Cobb-Douglas function, with energy, capital, labor as factors (see Keen et al., 2019, for the Energy-Augmented Cobb Douglas Production Function, EACDPF).[12]

[11] This model can be extended to multivariate z and z_1 (see below) to include other GHG, or wastes.

[12] In an analysis at corporate level, this EACDPF modelling has to be avoided. Indeed, if, at the origin, the carbon is a "public good" with price zero, this modelling with substitutability will lead to an infinite amount of energy to produce any y. As seen below, complementarity has to be introduced.

Profit maximization becomes:

$$
\begin{aligned}
& \max_{x_1,x_2,z_1,z} \pi y - \pi_c z - p_1 x_1 - p_2 x_2 - q z_1, \\
& \text{s.t.} \begin{cases} f_1(x_1,x_2;z_1) &= y \\ f_2(x_1,x_2;z_1) &= z. \end{cases} \\
\Leftrightarrow \; & \max_{x_1,x_2,z_1} \pi f_1(x_1,x_2;z_1) - \pi_c f_2(x_1,x_2,z_1) - p_1 x_1 - p_2 x_2 - q z_1 \\
& \text{s.t. } f_1(x_1,x_2;z_1) = y.
\end{aligned}
$$

This leads to another input allocation: $\left\{\hat{x}_1(y,p,\pi;\pi_c,q), \hat{x}_2(y,p,\pi;\pi_c,q), \hat{z}_1(y,p,\pi;\pi_c,q)\right\}$, and to a (potential) profit $\hat{P}(y,p,\pi;\pi_c,q)$. There is no production if $\hat{P}(y,p,\pi;\pi_c,q)$ is negative.

Example 3. *Carbon footprint*

The regulation for low carbon has implicitly selected specific forms of production functions of the type:

$$
\begin{cases} y &= g(x_1,x_2) + \mu \min(z_1, \gamma_1 x_1 + \gamma_2 x_2), \\ z &= f_2(x_1,x_2;z_1), \end{cases} \tag{3.16}
$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$. At the optimum, we have: $z_1 = \gamma_1 x_1 + \gamma_2 x_2$, and the profit becomes:

$$
\pi g(x_1,x_2) - \pi_c f_2(x_1,x_2;\gamma_1 x_1 + \gamma_2 x_2) + [(\mu\pi - q)\gamma_1 - p_1]x_1 + [(\mu\pi - q)\gamma_2 - p_2]x_2 \tag{3.17}
$$

The intuition behind these formulas is the following: the initial inputs x have not been disaggregated enough to account for the carbon used to get them. This approach tries to account for the carbon quality of each input by associating to each of them a so-called carbon (foot)print: γ_j, $\gamma_j > 0$, $j = 1,2$.

For instance, the footprint of "electricity" depends on its origin (nuclear plants, solar panels, or gas power plant). The footprint of the "labour" is not the same for a worker or a manager who regularly takes intercontinental flights. Profit formula (3.17) shows how input prices are adjusted for their carbon component by the terms $(\mu\pi - q)\gamma_j$. Even if the cost of input j is increased from p_j to $p_j + \gamma_j q$, the use of this input can be profitable if $(\mu\pi - q)\gamma_j > p_j$, even if $\mu\pi > q$ only.

Practical implementation of production functions of the type (3.16) can be found, e.g., in European Banking Authority (2020); Network for Greening the Financial System (2020); Boermans and Galema (2017).

A limitation of this approach is that it necessitates the knowledge of the production function itself, and of the exposure to carbon, either direct (called scope 1), or indirect (scope 2 and scope 3) (see, e.g., European Union, 2019; Novethic, 2020).

3.3.3 Change of technology

The previous derivation does not account for the possibility to change the technology to adjust for carbon valuation, that is to reshape the current productive structure. This possibility can be captured by making the technology parameter dependent. Let us replace (3.15) with:

$$\begin{cases} y &= f_1(x_1, x_2; z_1; \theta), \\ z &= f_2(x_1, x_2; z_1; \theta). \end{cases} \tag{3.18}$$

The optimization is then performed jointly with respect to x_1, x_2, z_1 and θ. This leads to an optimal technology $\hat{\theta}(y, p, \pi; \pi_c, q)$ and an optimal profit $\hat{\hat{P}}(y, p, \pi, \pi_c, q) = \hat{P}\left[y, p, \pi, \pi_c, q; \hat{\theta}(y, p, \pi, \pi_c, q)\right]$.

By considering Example 3 of carbon footprint, we see that the parametrization of the technology can be done either through the function g, the footprint coefficients, the function f_2, or the parameter μ. Typically, the firm can change the carbon quality of the inputs—making them more costly—but could also increase its added value by employing a more efficient production process. The firm can also decide to diminish or treat its carbon emissions: this is a costly change in production function f_2, ceteris paribus, that can be compensated by decreases in direct carbon emissions z.

This production function, that includes technology changes, can be given a "portfolio" interpretation. At the optimum the producer has chosen the best combination of inputs x_1, x_2 and technology θ, i.e., the best allocations associated with the (exogenous) prices and demand level y. In this respect the new prudential supervision has to follow not only the final profit of the firm, but also the changes in these allocations, including the changes in technology. This is in line with the recent prudential supervision for hedge funds, that follows not only the hedge funds returns, but also their portfolio allocations.

In what precedes, we assume that prices and demand are exogenous for the firms (prices evolve due to exogenous taxes, or penalties, and the firm does not account for the future evolution of demand). The analysis could be extended to account for a modification of the selected technology due to a change of demand by consumers, that become more sensitive to environmental issues, say. We consider also each firm as an autonomous entity. Thus we do not take into account the chain value and their impact on potential cascades of default. This would necessitate the knowledge of an input-output network (see Cahen-Fourot et al., 2020, 2021, for an attempt in this direction). This is out of the scope of this paper.

3.3.4 Progressive Change of Technology

The transition is induced by a dynamic change in prices, π_{ct} and q_t, which can for instance reflect the introduction of carbon taxes (see, e.g., Nordhaus, 2014; Gollier, 2018; King et al., 2019; Bolton et al., 2020, for estimates of the social cost of carbon), or new rules for eligible collateral that would penalize brown assets (see, e.g., Matikainen et al., 2017; Papoutsi et al., 2020). Accordingly, we now consider a progressive change in π_{ct} and q_t to adjust for the underpricing of the externalities, the other characteristics (y, p, and π) being crystallized. This provides dynamic signals to producers about which inputs and technologies are carbon intensive, and induce the firms to move to low carbon technologies.[13] These price changes are considered exogenous for the producer and for the financial supervisor. Then the technology $\hat{\theta}_t$ will change progressively over time, as well as the underlying profit $\hat{\hat{P}}_t$. With a strict definition of default, this will induce the end of the production (default), if there exists h such that $\hat{\hat{P}}_{t+h} < 0$, $h = 1, \dots, H$. However, this definition of default can be too severe, since losses at some dates could be compensated by positive profits later on. A transition of technology means that, at date t, a standard firm becomes *de facto* a type of "start up" that has to innovate. The short-run profits of a start up are usually negative, and may become positive in the long run if the firm's transition is successful. To account for this effect, we can consider a long-run definition of default based on the cumulated profit $CP_{t+H} = \sum_{h=1}^{H} CP_{t+h}$. This cumulated profit can be positive, if the firm is successful in the long run, and it is negative, otherwise. There is default if CP_{t+H} is negative.

The profit $\hat{\hat{P}}_{t+h}$ is a nonlinear function of current and past carbon prices, due to the optimal

[13]The introduction of carbon prices may not be sufficient to steer the economic system towards a low carbon transition. Other actions have been proposed, such as the creation of a green bank specialized in green loans (e.g., the Green Investment Bank created in 2012 in UK), green quantitative easing (de Grauwe, 2019), the introduction of differentiated reserves, depending on the destination sector of lending (Rozenberg et al., 2013; Campiglio, 2016), or proactive fiscal policies.

choice of technology and input allocations.

Let us now consider progressive changes of prices π_{ct} and q_t, these prices staying close to crystallized prices, π_c^* and q^*, say. A Taylor expansion leads to:

$$\begin{aligned} \hat{\hat{P}}_t &= \hat{P}\left[y, p, \pi, \pi_{ct}, q_t; \hat{\theta}(y, p, \pi; \pi_{ct}, q_t)\right] \\ &\simeq \hat{\hat{P}}^* + \frac{\partial \hat{P}^*}{\partial \pi_c}(\pi_{ct} - \pi_c^*) + \frac{\partial \hat{P}^*}{\partial q}(q_t - q^*) + \frac{\partial \hat{P}^*}{\partial \theta}\left[\frac{\partial \hat{\theta}^*}{\partial \pi_c}(\pi_{ct} - \pi_c^*) + \frac{\partial \hat{\theta}^*}{\partial q}(q_t - q^*)\right], \end{aligned} \tag{3.19}$$

where the upper $*$ index means that the derivatives are taken at y, p, π, π_c^*, q^*. Therefore, locally, we can apply (3.19) for any date $t+h$, and also to the cumulated (optimal) profit $CP_{t+H} = \sum_{h=1}^{H} \hat{\hat{P}}_{t+h}$. We get:

$$\begin{aligned} CP_{t+H} = H\hat{\hat{P}}^* \quad &+ \quad \left(\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial \pi_c}\right)\sum_{h=1}^{H}(\pi_{c,t+h} - \pi_c^*) \\ &+ \quad \left(\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta}\frac{\partial \theta^*}{\partial q}\right)\sum_{h=1}^{H}(q_{t+h} - q^*). \end{aligned} \tag{3.20}$$

We can use this expansion to translate the ESG risk due to the changes in prices into prudential risks, i.e., we can measure the long-run sustainability of this business model.

As mentioned earlier we will consider a progressive change of carbon prices, following Carney (2016)'s recommendation:[14] "a too rapid movement towards a low-carbon economy could materially damage financial stability, [...] destabilize markets, crystallize losses". More precisely, to evaluate the long-run uncertainty on the cumulated profit, we introduce ULR dynamics for prices π_{ct} and q_t. The difference with Subsection 3.2 is that these dynamics, close to a constant, have to be nonstationary in order to account for the transition between two economies.[15] This can be done in different ways, for instance by considering stochastic logistic transition based on the time discretization of:

$$d\tilde{y}(\tau) = k\tilde{y}(\tau)\{c - \tilde{y}(\tau)\}d\tau + \eta dW(\tau), \quad k > 0,\, c > 0,\, \eta > 0,$$

[14] See also Authority of Prudential Control and Resolution (ACPR) (2021), p.4: "Les modèles utilisés par les banques pour quantifier les risques ne sont pas adaptés pour intégrer des évolutions très lisses des variables macroéconomiques et financières sur longue période".

[15] This nonstationarity is sometimes called "non equilibrium model" in the literature (see, e.g., Bolton et al., 2020, p.44).

or on a Gaussian autogressive process:

$$y_t = \rho_H y_{t-1} + \sqrt{\rho_H^2 - 1}\, \varepsilon_t, \rho_H > 1,$$

where ρ_H tends to 1^+ when H tends to infinity. This latter process can be deduced by a time deformation of an Ornstein-Uhlenbeck process.

It can be shown that if the prices follow a logistic transition from initial zero prices (the carbon was not priced at the initial date), price changes are small at the beginning and end of the transition, but reach a maximum in between. The speed of the transition has a direct impact on this maximum. This can be the moment of the largest required changes to which corporates will not adjust and then will default. This is a tipping point in which the change can be irreversible for the firm.

Let us consider the expansion (3.20). We get:

$$\frac{1}{H} CP_{t+H} = \hat{\hat{P}}^* + B \frac{1}{H} \left[\sum_{h=1}^{H} (\pi_{c,t+h} - \pi_c^*), \sum_{h=1}^{H} (q_{t+h} - q^*) \right]', \tag{3.21}$$

where B is the row vector of multipliers, that is:

$$B = \left[\frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta} \frac{\partial \theta^*}{\partial \pi_c}, \quad \frac{\partial \hat{P}^*}{\partial \pi_c} + \frac{\partial \hat{P}^*}{\partial \theta} \frac{\partial \theta^*}{\partial q} \right].$$

If the sequence of prices is an ultra long-run process:

$$(\pi_{c,t+h}, q_{t+h})' = \tilde{Y}(h/H), \tag{3.22}$$

where $(\tilde{Y}(\tau))$ is a bivariate diffusion process, we get the approximation (for large H):

$$\frac{1}{H} CP_{t+H} \simeq \hat{\hat{P}}^* + B \left[\int_0^1 \tilde{Y}(u) du - \begin{pmatrix} \pi_c^* \\ q^* \end{pmatrix} \right]. \tag{3.23}$$

Formula (3.23) can be used to analyze the uncertainty on the cumulated profit at any intermediate date, then to propose a required capital profile, or to evaluate the probability of default (considering that a default takes place when CP_{t+H} is negative). For instance, the probability of default, at a date $t+h$, is:

$$\begin{aligned} PD_{t+h} &= \mathbb{P}_{t+h}[CP_{t+H} < 0] \\ &\simeq \mathbb{P}_{t+h}\left[\hat{\hat{P}}^* + B\left[\int_0^1 \tilde{Y}(u)du - \begin{pmatrix} \pi_c^* \\ q^* \end{pmatrix}\right] < 0\right]. \end{aligned}$$

If $(t+h)/H \simeq h/H \simeq \gamma$ for large H, we get:

$$PD_{t+h} \simeq \mathbb{P}\left[\hat{\hat{P}}^* + B\left[\int_0^\gamma \tilde{Y}(u)du - \begin{pmatrix} \pi_\gamma^* \\ q^* \end{pmatrix}\right] + B\int_\gamma^1 \tilde{Y}(u)du < 0 \middle| \underline{\tilde{Y}(\gamma)}\right],$$

where $\underline{\tilde{Y}(\gamma)} = (-\underline{\pi_{c,t+h}}, \underline{q_{t+h}})'$ is the information on prices available at date $t+h$.

4 Implementation

Let us now illustrate how the approach can be implemented. We consider the framework of Subsections 3.2 and 3.3, which mixes short- and long-run factors.

4.1 A Multistep Approach

The approach follows the steps below:

1. Define the number(s) of short- and long-run factors and the (nonlinear) function g, in (3.2).

2. Define the horizon $T = t+H$.

3. Specify the distribution of the short-run component(s), and the dynamics of the long-run component(s).

4. Deduce, by simulation, or by applying a first-order expansion, the conditional distribution of CP_{t+H} at date $t+h$. In simple models (see Subsection 4.2), this conditional distribution is known under closed form. Otherwise, it is obtained by simulation.

5. Fix the valuations $X^*_{t+h,t+H}$ from this conditional distribution, for instance by a mean-variance scheme (see other possibilities in Subsection 2.2).

6. Fix the control parameters δ, r defining the exponential profile and the supervisory discount rate, respectively.

7. Compute the sequences RCC and RC by formula (2.6).

8. Compute the short-run required capital rc using (3.1).

4.2 Stochastic Volatility Model with ULR Volatility

Let us first consider a profit corresponding to an evolution with long-run stochastic volatility defined by:

$$P_t \quad = \quad g(y_{s,t}, y_{l,t}) = a + b y_{l,t} + \sqrt{y_{l,t}}\, y_{st},$$

where y_{st} is a strong white noise, $y_{l,t} = \tilde{y}(t/T)$ an ULR stochastic volatility. The modelling above includes a risk premium with an effect of the long-run volatility on the conditional mean. By applying (3.8)-(3.10) and noting that $\mathbb{E}(y_{s,t}) = 0$, we get:

$$\frac{1}{H} CP_{t+H} \sim a + b \int_0^1 \tilde{y}(u) du. \tag{4.1}$$

In the long run, the effect of $\sqrt{y_{l,t}} y_{s,t}$ can be neglected, but the randomness associated with the long-run risk premium has to be taken into account.

Next we apply the mean-variance scheme to fix the intermediate valuations. Since $y_{l,t}$ is a volatility, we posit a dynamics of $y_{l,t}$ based on a continuous time CIR process (Cox et al., 1985).[16] Specifically, we take $y_{l,t} = \tilde{y}(t/H)$, where the continuous-time dynamics of $\tilde{y}(\tau)$ is defined by the diffusion equation:

$$d\tilde{y}(\tau) = K\{\theta - \tilde{y}(\tau)\} d\tau + \eta\, \tilde{y}^{1/2}(\tau) d\tilde{W}(\tau), \tag{4.2}$$

where K, θ, η are parameters, with $K > 0$, $\theta > 0$, $\eta > 0$, and where the Feller condition holds ($2K\theta > \eta^2$), and $\tilde{W}$ is a Brownian motion.

Since the CIR process is a special case of affine process, the conditional log-Laplace transform of its future cumulated values is a linear affine function of its current value. Then by considering the second-order Taylor expansion of this log-Laplace transform, we deduce that the associated

[16] An alternative would be to assume that $\log \tilde{y}$ follows an Ornstein-Uhlenbeck process.

conditional mean and variance are linear affine as well. Thus we have:

$$\mathbb{E}\left[\int_\gamma^1 \tilde{y}(u)du|\underline{\tilde{y}(\gamma)}\right] = m_1(1-\gamma)\,\tilde{y}(\gamma)+m_0(1-\gamma), \tag{4.3}$$

$$\mathbb{V}ar\left[\int_\gamma^1 \tilde{y}(u)du|\underline{\tilde{y}(\gamma)}\right] = \sigma_1(1-\gamma)\,\tilde{y}(\gamma)+\sigma_0(1-\gamma), \tag{4.4}$$

where m_1, m_0, σ_1, and σ_0 are functions of the time-to-maturity and of the parameters characterizing the dynamics (4.2) of the CIR process.

When H is large:

$$\begin{aligned}\frac{1}{H}CP_{t+H} &= a+b\int_0^1 \tilde{y}(u)du \\ &= \frac{H-h}{H}a+\frac{1}{H}CP_{t+h}+b\int_{h/H}^1 \tilde{y}(u)du.\end{aligned}$$

This implies, for the mean-variance scheme:[17]

$$\begin{aligned}X^*_{t+h,t+H} &\simeq \mathbb{E}_{t+h}(CP_{t+H})-\frac{A}{2}\mathbb{V}ar_{t+h}(CP_{t+H}) \\ &\simeq (H-h)a+CP_{t+h}+ \\ &\quad bH\left(m_0\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_0\left(1-\frac{h}{H}\right)\right)+ \\ &\quad bH\left(m_1\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_1\left(1-\frac{h}{H}\right)\right)y_{l,t+h}.\end{aligned} \tag{4.5}$$

Hence, $X^*_{t+h,t+H}$ can be approximated by an affine combination of the current cumulated profit (CP_{t+h} as of date $t+h$) and of the current value of the long-run component, with coefficients depending on the time-to-maturity.

Next, we can apply (2.6), the recursive formula defining the RCC. For instance, with $\delta=1$ and $r=0$, we get:

$$\begin{aligned}RCC_{t+h} &= X^*_{t+h,t+H}/(H-h) \\ &= a+\frac{CP_{t+h}}{H-h}+\frac{bH}{H-h}\left[m_0\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_0\left(1-\frac{h}{H}\right)\right] \\ &\quad +\frac{bH}{H-h}\left[m_1\left(1-\frac{h}{H}\right)-\frac{A}{2}bH\sigma_1\left(1-\frac{h}{H}\right)\right]y_{l,t+h}.\end{aligned} \tag{4.6}$$

[17] The minus sign in front of A results from the fact that X is, here, a gain. See the discussion at the end of (ii) on page 9.

For large h, and using the ULR property of component $y_{l,t}$, we obtain:

$$\begin{aligned} RCC_{t+h-1} &\sim RCC_{t+h}, \\ P_{t+h+1} &\sim a + by_{l,t+h} + \sqrt{y_{l,t+h}}\, y_{s,t+h+1}. \end{aligned}$$

Following the approach proposed in (3.1), the short-run required capital $rc_{t+h}(\alpha)$ is approximately given by:

$$\mathbb{P}_{t+h}\left[a + by_{l,t+h} + \sqrt{y_{l,t+h}}\, y_{s,t+h+1} + rc_{t+h}(\alpha) > RCC_{t+h}\right] = 1 - \alpha,$$

or

$$rc_{t+h}(\alpha) = RCC_{t+h} - a - by_{l,t+h} - q(\alpha)\sqrt{y_{l,t+h}}, \tag{4.7}$$

where $q(\alpha)$ is the α-quantile of the distribution of the short-run component.

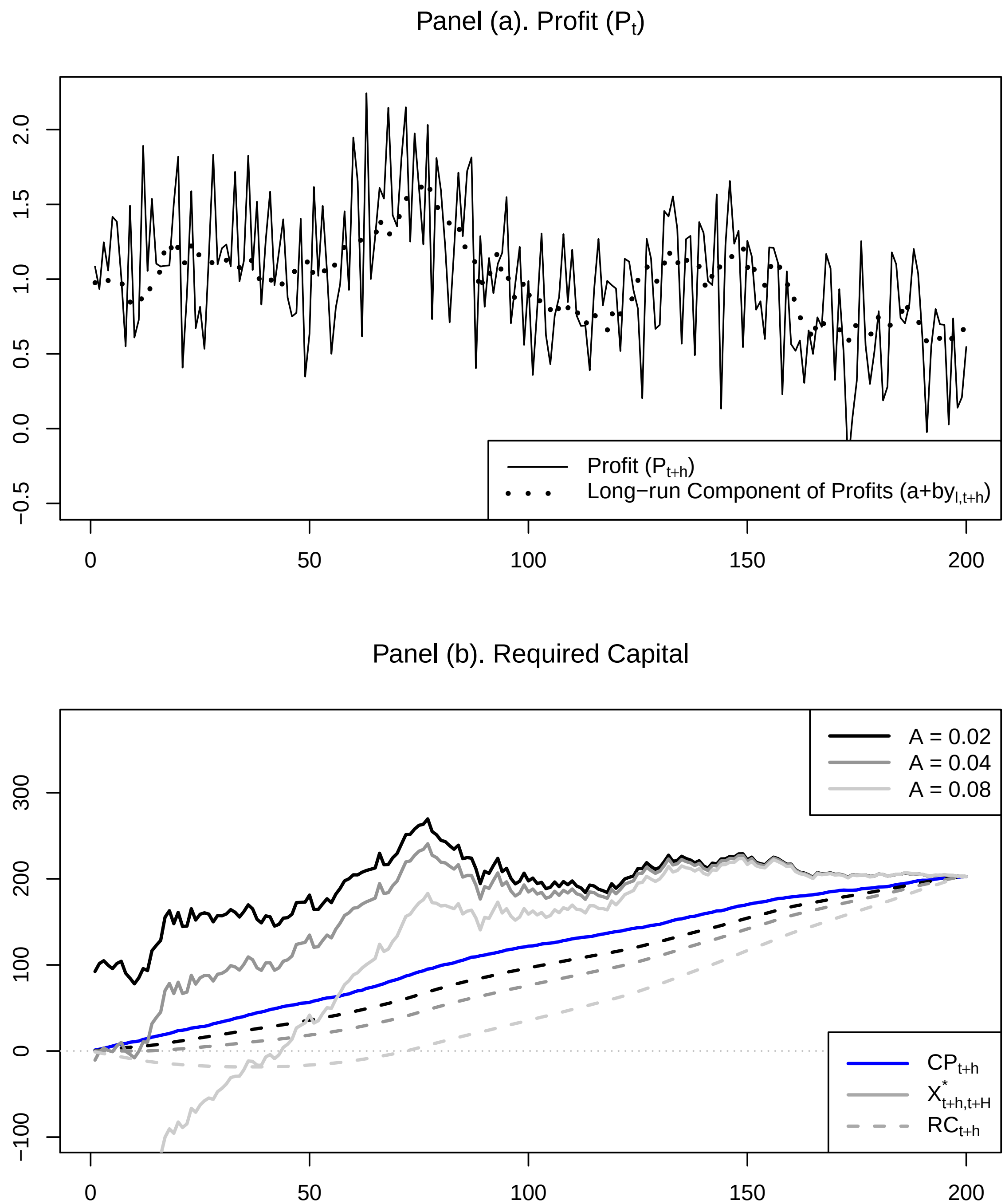
This approach requires the knowledge of the distribution of the short-run component, of the parameters of the underlying CIR process, and an approximation of the ULR component. These estimation and filtering issues are out of the scope of the present paper.

Figure 4 displays simulated paths based on the framework described in the present subsection. Specifically, we consider the process of profits P_t described by (4.1), with $a = -3$ and $b = 1$. Moreover, the specification of the CIR process (4.2) is as follows: $K = 0.05$, $\eta = 0.45$, and $\theta = 4$. Finally, $y_{s,t} \sim i.i.d.\, \mathcal{N}(0, \sigma^2)$, with $\sigma = 0.2$. The valuation X^* is based on a mean-variance scheme, and is approximated by (4.5). We consider different values of A, reflecting different degrees of absolute risk aversion. Using $\delta = 1$ and $r = 0$, the computation of the Required Capitals Calls is based on (4.6).

We obtain lower RCs for larger values of A. The figure also illustrates the convergence of required capitals RC_{t+h} to the valuation $X^*_{t+h,t+H}$ when h goes to H.

5 Concluding Remarks

The objective of this paper is to explore the prudential supervision for long-run risk by considering a sequence of required capital calls. We first consider pure long-run risks. Then we extend the approach to a joint computation of reserves for short- and long-run risks, and take the example of those risks underlying the transition to a low-carbon economy. The latter example highlights the



Notes: We consider a process of profits P_t described by (4.1) (with $a = -3$ and $b = 1$). The process followed by the long-run component $y_{l,t}$ is based on (4.2); specifically $y_{l,t} = \tilde{y}(t/H)$. The CIR process (4.2) is parameterized with $K = 0.05$, $\eta = 0.45$, and $\theta = 4$. The short-run component $y_{s,t}$ is a Gaussian white noise of standard deviation 0.2. The valuation X^* is based on a mean-variance scheme, and is approximated by (4.5). The computation Required Capital Calls is based on (4.6) (using $r = 0$ and $\delta = 1$). Parameter A is the risk aversion of the mean-variance scheme (equation 2.8).

Figure 4: Simulated required capital RC_{t+h}

importance of updating the knowledge of carbon inputs and outputs involved in the production process, which, itself, requires a complete and transparent accounting framework.[18] This is currently under discussion in the context of the prudential supervision to a low carbon economy, for which a first report for December 2021 has been announced with relevant Guidelines scheduled for June 2025 (European Banking Authority, 2019, 2020), and the recently-published results of the French pilot exercise 2020 (Authority of Prudential Control and Resolution (ACPR), 2021).

The determination of a required capital profile for long-run risk is an evolving field, and the implementation of such a supervision suppose data, standardizations and taxonomies. For the transition to a low carbon economy, the standardizations concern the precise definitions of homogenous classes of goods (inputs and outputs), of the GHG, of the production functions, of the reporting frequency (with the objective of one year, not yet fulfilled), of the industrial sectors,[19] of the subset of sustainable sectors (Husson-Traore, 2019), of the control of the reporting quality, and of the horizon of analysis.

Important questions of coherency between micro analysis of prudential supervision and dynamic macro models introduced for macro predictions have also to be solved. Let us give an example of such questions: "prices" π_c are an instrument of economic policy, that can be used at the individual level of the firm. During the transition, the costs of carbon issuing do not have to be the same for the different industrial sectors, since these sectors do not have the same vulnerability to—or effect on—climate changes. The supervision has to be "proportionated, tailored for different business models around the sector, recognizing that the zero failure is neither desirable, nor realistic" (Carney, 2015). This is not compatible with the macro models, where "the carbon prices should be equalized in every sector and country" [Nordhaus (2019), p.2002, see also Weitzman (2014)]. Indeed standard macro models are assuming a representative individual and do not really account for firm heterogeneity [see Nordhaus (2014) for a description of the DICE (Dynamic Integrated Model of Climate and the Economy) model, and Keen (2019) for a critical view].

Even if it is a prudential-supervision-oriented paper, the developed approach provides measures of long-run risks, such as environmental transition risks, that can be used for other purposes. They could serve as a basis to construct ratings for (environmental) long-run risks, and then labels for in-

[18] This is the analogue of the reporting of Securities and Exchange Commission's (SEC) Form 13F.

[19] A dozen of nomenclatures are currently used. They are not exempt from conflicting views. They include the International Standard Industrial Classification of all Economic Activities (ISIC) from the United Nations, the Nomenclature des Activités Commerciales et Économiques (NACE) for the European Union, the Harmonized System (HS) by the World Customs Organization, the Global Industry Classification Standard (GICS) created by Standard and Poor's and Barra, the Industry Classification System (ICS) created by Dow Jones and FTSE.

vestment responsible (IR) funds, ESG funds, green bonds (see HSBC, 2014; Fender et al., 2019),[20] sustainability linked bonds (SLB), or syndicated loans (Ehlers et al., 2021). These applications are beyond the scope of this paper.

The approach of required capital developed in our paper does not explain how different parameters such as the rate δ of the exponential profile, the "contractual" rate r, or the sequence of valuations $X^*_{t+h,t+H}$, will be fixed. In the context of the transition to a low-carbon economy, the choice of these control parameters has an impact on credit costs, credit granting, and credit allocation between firms, as well as on the term structure of risk/return in low carbon investment. Hence, such economic-policy tools can modify the role of public regulators in credit issuance and financing (see, e.g., the postulate of money multiplier theory, Mishkin, 2001). This multidimensional tool is much more flexible than the standard scalar monetary policy instrument, namely the reference interest rate. Currently, an "optimal" choice of these parameters is not directly in the mandate of the prudential supervision, nor in that of the central banks.

[20]The EU Ecolabel criteria are expected to be included in the European regulation in 2021.

A Required Capital and Value-at-Risk

Let us briefly review the standard approach for required capital. We keep the same notations as in the main text. That is, X_{t+H} is a (positive) loss, and $-X_{t+H}$ is a gain. The required capital at $t+h$ is chosen to bound the conditional probability of loss at $t+H$. When RC is not remunerated, we have:

$$\mathbb{P}_{t+h}[-X_{t+H} + RC_{t+h} < 0] = \alpha, \tag{a.1}$$

where α is the regulatory critical level and $\mathbb{P}_{t+h}$ is the conditional probability given the information available at date $t+h$. Equation (a.1) rewrites:

$$\begin{aligned} & \mathbb{P}_{t+h}[-X_{t+H} + RC_{t+h} > 0] = 1-\alpha \\ \Leftrightarrow \quad & \mathbb{P}_{t+h}[X_{t+H} < RC_{t+h}] = 1-\alpha \\ \Leftrightarrow \quad & RC_{t+h} = q_{t+h}(1-\alpha), \end{aligned} \tag{a.2}$$

where q_{t+h} is the conditional quantile of X_{t+H} at date $t+h$, usually called Value-at-Risk (VaR).

This VaR measure of required capital depends on the following policy instruments: (i) the horizon H; (ii) the updating frequency; (iii) the information available at date $t+h$; and (iv) the specification of the conditional distribution.

Example 4. *Arbitrage portfolio*

Denote by p_t the value of a stock with geometric returns such that:

$$y_{t+1} = \log p_{t+1} - \log p_t = \mu + \sigma u_t,$$

where the u_t's are independent identically normally distributed. This simple dynamics underlies the Black-Scholes approach.

We have:

$$\log p_{t+H} - \log p_{t+h} = \mu(H-h) + \sigma\sqrt{H-h}\,U, \quad \textit{where } U \sim N(0,1).$$

Consider a portfolio invested at date t in one unit of the stock and a short sell of p_t in riskfree asset. This is an arbitrage portfolio with zero value at date t. With zero riskfree rate, we *have, at date* $t+H$*:* $X_{t+H} = p_t - p_{t+H}$*. We deduce:*

$$
\begin{aligned}
& \mathbb{P}_{t+h}(p_t - p_{t+H} < RC_{t+h}) \\
= \; & \mathbb{P}_{t+h}(p_{t+H}/p_t > 1 - RC_{t+h}/p_t) \\
= \; & \mathbb{P}_{t+h}[\log(p_{t+H}/p_t) > \log(1 - RC_{t+h}/p_t)] \\
& \text{(note that } RC_{t+h} < p_t\text{, since the maximum loss is } p_t\text{).} \\
= \; & \mathbb{P}_{t+h}\left[U > \{\log[(p_t - RC_{t+h})/p_{t+h}] - \mu(H-h)\}/\sigma\sqrt{H-h}\right].
\end{aligned}
$$

For this to be equal to $1-\alpha$*, we need to have:*

$$RC_{t+h} = p_t - p_{t+h}\exp\left[\mu(H-h) + q(\alpha)\sigma\sqrt{H-h}\right], \tag{a.3}$$

where $q(.)$ *is the quantile function of the standard normal distribution. In practice* α *is small, and* $q(\alpha)$ *is negative.*

This example is not purely theoretical, especially when considering the balance sheet of a bank. Indeed, the updating of such balance sheet is largely due to the issuance of new credits. Newly issued credits change the balance sheet by introducing the same value of the initial balance in both asset and liability sides, implying a zero initial value of this portfolio change.

Let us now discuss some drawbacks of this standard approach when applied to long-run risk, that is, if $H-h$ is large.

First, if $H-h$ is large, we get: $RC_{t+h} \simeq -p_{t+h}\exp[\mu(H-h)]$, whenever $\mu > 0$. Thus, the pure risk σ is not taken into account in the definition of the required capital. In other words, in the model of random walk with drift for the return, the pure risk is implicitly assumed diversified in the long run. Moreover the value RC_{t+h} tends to $-\infty$, that means that the investor could profit of this diversification to invest $-RC_{t+h}$ in risky asset.[21]

Second, and symmetrically, if μ is negative, RC_{t+h} tends to p_t when $H-h$ tends to infinity. The standard VaR approach then requires a perfect hedge of the stock at any date to cover the short sell, and this RC is independent of the critical level α.

Third, let us now consider how the potential RC (without taking into account the nonnegativity constraint) depends on time-to-maturity. Two cases have to be distinguished. For an adverse

[21] In fact the current supervision assumes a nonnegative required capital, and then this natural incentive to take more risk does not exist.

evolution ($\mu < 0$), RC_{t+h} is an increasing function of time-to-maturity. Thus there is no incentive to invest in long-run risk. Next, for a positive evolution ($\mu > 0$), RC_{t+h} can be first increasing, then decreasing. Indeed in the short run ($H-h$ small) the volatility dominates the tendency.

Fourth, this approach assumes the knowledge, or at least an approximation, of the conditional distribution of p_{t+H} (or of the returns). This is not realistic for the cases we are interested in, where the associated long-run risks are not traded on liquid organized markets.

All in all, "(whereas) existing modelling instruments allow for a good measurement of market risk, [...] over relatively small time intervals, these (VaR) approaches may have severe deficiencies if they are routinely applied to longer time periods" (Embrechts et al., 2005).

B Long-run Approximation of Cumulated Profit

We want to show that $\frac{1}{H}\sum_{h=1}^{H} g\left[y_{s,t+h};\tilde{y}\left(\frac{t+h}{H}\right)\right]$ converges in probability (when $H \to \infty$) to

$$\int_0^1 G_s(\tilde{y}_\tau)d\tau, \quad \text{where } G_s(\tilde{y}) = \mathbb{E}_s[g(y_s,\tilde{y})],$$

and $\mathbb{E}_s$ denotes the expectation with respect to y_s given $\tilde{y}$.

B.1 Specific case

Let us first consider the case where

$$g\left[y_{s,t+h};\tilde{y}\left(\frac{t+h}{H}\right)\right] = g_1\left(y_{s,t+h}\right) g_2\left(\frac{t+h}{H}\right).$$

We have:

$$\frac{1}{H}\sum_{h=1}^{H} g_1\left(y_{s,t+h}\right) g_2\left(\frac{t+h}{H}\right) = \mathbb{E}(g_1)\frac{1}{H}\sum_{h=1}^{H} g_2\left(\frac{t+h}{H}\right) + \frac{1}{H}\sum_{h=1}^{H}\left[g_1\left(y_{s,t+h}\right) - \mathbb{E}(g_1)\right] g_2\left(\frac{t+h}{H}\right).$$

Thanks to the convergence of the Riemann sum to stochastic integrals, the first term converges in probability to $\mathbb{E}(g_1)\int_0^1 g_2(\tilde{y}_\tau)d\tau = \int_0^1 G_s(\tilde{y}_\tau)d\tau$. It remains to check that the second term converges to zero in probability.

Let us introduce the notation

$$X_{H,h} = \left[g_1\left(y_{s,h}\right) - \mathbb{E}(g_1)\right] g_2\left(\frac{h}{H}\right).$$

It is a zero-mean double stochastic array to which we can apply a suitable weak law of large numbers (LLN). Let us consider the weak LLN of de Jong (1995, 1998).[22] We first check that $X_{H,h}$ is a L_1-mixingale array; more precisely, that

$$\left\|\mathbb{E}\left[X_{H,h}\left|\underline{y_{s,t-m}}, \underline{\tilde{y}\left(\frac{h-m}{H}\right)}\right.\right]\right\|_1 < c_{H,h}\psi(m),$$

for some $c_{H,h}$ and $\psi(m) \to 0$, when $m \to \infty$, where $\|\cdot\|_1$ denotes the L_1 norm.[23] Thanks to the independence of processes y_s and $\tilde{y}$, the left-hand-side term is equal to

$$\left\|\mathbb{E}\left[g_1\left(y_{s,h}\right) - \mathbb{E}(g_1)\left|\underline{y_{s,t-m}}\right.\right]\right\|_1 \times \left\|\mathbb{E}\left[g_2\left(\frac{h}{H}\right)\left|\underline{\tilde{y}\left(\frac{h-m}{H}\right)}\right.\right]\right\|_1.$$

Under mild regularity conditions, the second term of the product above is bounded by c (say). If the zero mean process $g_1\left(y_{s,h}\right) - \mathbb{E}(g_1)$ is a L_1-mixingale, given the stationarity, the first term of the product is dominated by $\psi_s(m)$, where $\psi_s(m) \underset{m\to\infty}{\to} 0$. Therefore, taking $c_{H,h} = c$ and $\psi(m) = \psi_s(m)$, we see that $X_{H,h}$ is a L_1-mixingale array.

Let us now check that we can apply the weak LLN of de Jong (Theorem 4 in de Jong, 1998). The first condition is as follows: for some sequence $B_H \geq 1$, and $B_H = o(H^{1/2})$, we have

$$\lim_{K\to\infty} \limsup_{H\to\infty} \frac{1}{H} \sum_{h=1}^{H} \left\|X_{H,h} \mathbb{1}_{\{X_{H,h} > KB_H\}}\right\|_1 = 0.$$

This condition is satisfied by taking $B_H = 1$, by using the fact that the distribution of $X_{H,h}$ does not depend on h, and that $\mathbb{E}|X_{H,h}| < \infty$.

The second condition is that, for all $K > 0$, $\lim_{H\to\infty} \frac{1}{H} \sum_{h=1}^{H} c_{H,h}\psi\left(KB_H^{-1}H^{1/2}\right) = 0$.

Since $c_{H,h} = c$ and $B_H = 1$, it is true as soon as $\psi\left(KH^{1/2}\right) \underset{H\to\infty}{\to} 0$, that is, if $\psi(m) = o(m^{-1/2})$, i.e., if the process $g_1(y_{s,h})$ (seen as a process in h), is a L_1-mixingale of size smaller than $-1/2$.[24]

[22] Theorem 4 in de Jong (1998) extends Theorem 1 of de Jong (1995) to the case of arrays.

[23] See de Jong (1995) (Definition 1) for a definition of (non-array) L_1-mixingales.

[24] A L_1-mixingale is said to be of size $-\beta$, $\beta > 0$, if the associated sequence $\psi(m)$ is such that $\psi(m) = O(m^{-\beta-\varepsilon})$ for some $\varepsilon > 0$.

This implies a condition on the dependence of process $g_1(y_{s,h})$. This condition is rather reasonable since, for instance, a stationary ARMA process based on an integrable martingale difference sequence is a L_1-mixingale of size $-\infty$.

Finally, by applying de Jong's theorem, we conclude that $\frac{1}{H}\sum_{h=1}^{H} X_{H,h}$ converges to 0 in the L_1 sense when $H \to \infty$, and, therefore, also in probability.

B.2 General case

In the general case, we can decompose function $g\left[y_{s,t+h};\tilde{y}\left(\frac{t+h}{H}\right)\right]$ into an (infinite) sum of product functions $g_1\left(y_{s,t+h}\right)g_2\left(\frac{t+h}{H}\right)$ or, alternatively, we can consider its first-order expansion (see equation 3.4):

$$g\left[y_{s,t+h};\tilde{y}\left(\frac{t+h}{H}\right)\right] \approx g\left[y_{s,t+h};y_l^*\right] + \frac{\partial g}{\partial y_l}\left[y_{s,t+h};y_l^*\right]\left[\tilde{y}\left(\frac{t+h}{H}\right) - y_l^*\right],$$

and we are back to the framework of a product of functions, as considered in Subsection B.1.

C Nonlinear Prediction Formulas for the CIR Process

Since it is an affine process, the CIR process admits an exponential affine Laplace transform, that is:

$$\begin{aligned} \psi(v;t,h) &= \log \mathbb{E}_t\left\{\exp\left[-v\int_t^{t+h}\tilde{y}(u)du\right]\right\} \\ &= C(v;h) - D(v;h)\tilde{y}_t. \end{aligned}$$

Closed-form solutions for functions C and D can be found, e.g., in Gouriéroux et al. (2021), Corollary 1.

Since the Taylor expansion of a log-Laplace transform is:

$$\log \mathbb{E}[\exp(-vZ)] \simeq -v\mathbb{E}(Z) + \frac{v^2}{2}\mathbb{V}arZ,$$

we deduce, using the notations of (4.3) and (4.4), that:

$$
\begin{aligned}
m_0(h) &= -\frac{\partial C(0;h)}{\partial v}, \quad m_1(h) = \frac{\partial D(0;h)}{\partial v}, \\
\sigma_0(h) &= \frac{\partial^2 C(0;h)}{\partial v^2}, \quad \sigma_1(h) = -\frac{\partial^2 D(0;h)}{\partial v^2}.
\end{aligned}
$$

Acronyms

ACPR: Autorité de Contrôle Prudentiel et de Résolution

BC: British Columbia

BIS: Bank for International Settlements

DICE: Dynamic Integrated Model of Climate and the Economy

EACDPF: Energy-Augmented Cobb-Douglas Production Function

EBA: European Banking Authority

ES: Expected Shortfall

EU: European Union

ESG: Environmental, Social and Governance

GHG: Greenhouse Gas.

GHGRP: Greenhouse Gas Emission Report

GICS: Global Industry Classification Standard

HCF: Hydrofluorocarbon

HLCCP: High Level Commission on Carbon Prices

HS: Harmonized System

HSBC: Hong-Kong Shanghai Banking Corporation

IAM: Integrated Assessment Model

ICBC: Industrial and Commercial Bank of China

ICS: Industry Classification System

ILS: Insurance Linked Security

IPCC: Intergovernmental Panel on Climate Change

IR: Investment Responsable (fund)

ISIC: International Standard Industrial Classification

NACE: Nomenclature des Activités Commerciales et Économiques

NGFS: Network for Greening the Financial System

OECD: Organisation for Economic Co-operation and Development

PFC: Perfluorinated Hydrocarbons

RCC: Required Capital Call

SEC: Securities and Exchange Commission

SLB: Sustainability Linked Bonds

UIC: Uniform Integrable in Cesaro (sense)

VaR: Value-at-Risk

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